**Barron’s Let’s Review Regents – Algebra I**

# Chapter 11: Sequences

## 11.1 Types of Sequences

A sequence is a list of numbers. It is sometimes possible to predict the next number on a list by examining the numbers before and detecting a pattern. The pattern could be adding a number to the term before, multiplying a number by the term before, or something even more complicated.

**Sequence Notation**

An example of a sequence is 3, 7, 11, 15, 19, …

The sequence is often named with the letter a. The individual elements of the sequence are named by the name of the sequence with a subscript identifying the element’s position in the list. For this list, . Sometimes instead of a subscript, the term number is put into parentheses, like with functions, like or .

**Arithmetic Sequences**

To find the value of , check to see if this is the kind of sequence where each number can be calculated by adding or subtracting the same thing from the previous number. When this happens, this sequence is called an *arithmetic sequence.*

In this example, since 3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15, and 15 + 4 = 19, this seems to be an arithmetic sequence. To continue the pattern,   
.

The sequence 40, 32, 24, 16 is also an arithmetic sequence. By subtracting 8 from each term, the next term is obtained. When each term is smaller than the previous term, it is a *decreasing* sequence. When each term is larger than the previous term, it is an *increasing* sequence.

**Geometric Sequences**

The sequence, 3, 6, 12, 24, 48, … is not an arithmetic sequence. 3 + 3 = 6, but 6 + 3 is not 12. In this sequence, each term is found by multiplying the previous term by 2. When this happens, the sequence is called a geometric sequence. The next term is . Since each term is larger than the previous term, this is an increasing geometric sequence. A sequence like 160, 80, 40, 20, … is a decreasing geometric sequence, since each term is ½ the previous term.

### Check Your Understanding of Section 11.1

1. Multiple-Choice
2. In the sequence , what is the value of ?  
   (2) 3
3. What type of sequence is 3, 7, 11, 15, … ?  
   (1) Increasing arithmetic
4. What type of sequence is 25, 18, 11, 4, … ?  
   (2) Decreasing arithmetic
5. What type of sequence is 4, 8, 16, 32, …. ?  
   (3) Increasing geometric
6. What type of sequence is 1215, 405, 135, 45, … ?  
   (4) Decreasing geometric
7. What is the next number in the sequence 8, 13, 18, 23?  
   (3) 28
8. What is the next number in the sequence 14, 6, -2, -10?  
   (2) -18
9. What is the next number in the sequence ?  
   (1)
10. A sequence begins with 2, 6, If this is a geometric series, what is the next number?  
    (4) 18
11. What type of sequence is ?  
    (3) Increasing geometric

## 11.2 Closed Form Defined Sequences

A recursive rule is useful for finding the next term of a sequence. If, however, you need to find the 100th term of a sequence, the recursive rule is not very convenient as it would require finding all 100 terms. A *closed form* definition of a sequence is a formula that relates the position of the number in the sequence to the number in that position. For arithmetic and geometric sequences, there is a short way to create the closed form definition.

**Listing a Sequence from the Closed Form Definition**

Like a function, the closed form definition takes the position of the number on the list as an input, and it outputs the number that goes into that position.

If the closed form definition of a sequence is   
, then the sequence can be created by substituting 1, 2, 3, and so on for *n*. This formula does not require looking at the previous term.

So, the sequence begins 10, 13, 16, …

**Example 1**

Finding the Closed Form Definition of an Arithmetic Sequence

The sequence 8, 11, 14, 17, … is an arithmetic sequence. Since 11 = 8 + 3, 14 = 8 + 3 2, the closed form definition for this sequence is:

**Math Facts**

The closed form formula for describing an arithmetic sequence is where is the first term in the sequence and *d* is the difference between any two consecutive terms.

Finding the Closed Form of a Geometric Sequence

2, 10, 50, 250, 1,250, … is a geometric sequence since each term is equal to the previous term multiplied by 5.

In general, the nth term will be .

**Math Facts**

The closed form formula for describing a geometric sequence is where is the first term and *r* is what you have to multiply by the previous term to get the next term.

**Example 4**

What is the tenth term of the geometric sequence that begins with 3, 12, 48, 192, … ?

Since the first term is 3 and each term is 4 times the previous term, and . The closed form formula for the terms of this sequence, then, is   
. For n = 10, this becomes

**Closed Form Definitions of Sequences for Real-World Scenarios**

If a real-world scenario has numbers that form an arithmetic sequence, the closed form definition can be used to describe the real-world scenario.

Suppose there is a swimming pool being filled by a hose according to an arithmetic progression. It has 4 inches of water after the first hour, then seven inches after at the end of the second hour, and then 10 inches of water at the end of the third hour. Finding a closed form rule to describe the height of the water in the pool after n hours is the same as finding the rule for the sequence 4, 7, 10, 13, …

Since and , the formula is   
, which can be simplified to   
.

**Closed Form Definitions of Sequences for Picture Patterns**

If a picture pattern models an arithmetic sequence, the closed form equation will describe that sequence too.

### Check Your Understanding of Section 11.2

1. Multiple-Choice
2. What are the first three terms of the sequence generated by the definition   
   ?  
   **(2) 2, 5, 8**
3. What is the fifth term of the sequence by the definition ?  
   **(3) -6**
4. What is the third term of the sequence generated by the definition ?  
   **(4) 48**
5. What is the fifth term of the sequence generated by the definition ?  
   **(3)**
6. What definition would produce the sequence 4, 13, 22, 31, … ?  
   **(2)**
7. What definition would produce the sequence 4, 12, 36, 108, … ?  
   **(1)**
8. Which expression could be used to find the 20th term of the sequence 5, 9, 13, 17, 21, … ?  
   **(2) 5 + 4(19)**
9. Which expression could be used to find the 20th term of the sequence 5, 15, 45, 135, …?  
   **(3)**
10. A person has saved $20. Each week he adds $7 to the savings. Which definition can be used to generate a sequence of the amount of money saved at the beginning of each week?  
    **(4)**
11. What picture will generate a sequence of the number of hexagons in each picture?  
    **(1)**
12. *Show how you arrived at your answers*.
13. Liam says that the sequence 7, 12, 17, 22, … can be written as . Cecilia says it can be written as   
    . Who is right? Explain.  
      
    **Both solutions work, but Cecilia is using the recursive form, while Liam is using the Closed Form. The recursive form is not very convenient for the 100th item in a sequence because items 1-99 would have be computed first. In the closed form, it is not necessary to compute items 1-99 to compute the 100th term in the sequence.**
14. What is the 100th term of the sequence 26, 19, 12, 5, …?
15. A folded sheet of paper is miles thick. If a sheet of paper is folded, the thickness doubles. (a) Create a definition that will generate a sequence where **the first number is**  and the next number is always double the previous number. (b) Use the definition to determine the height of the folded paper after 38 folds. Will this reach the moon (240,000 miles away)?  
      
    **(a)   
    (b) 216,917.5 miles  
    No. This will not reach the moon.**  
    **The first number is , not The book answer is incorrect because it uses , not .**
16. The first terms of sequence a are 1,000; 1,200; 1,400; 1,600; …. The first term of sequence b are: 1, 2, 4, 8, … Which is greater,   
     or ?  
      
     **is greater than .**
17. To find the 100th term of the sequence 5, 8, 11, 14, …, would it be faster to use the closed form definition or the recursive definition of the sequence?  
      
    **Closed Form:  
    , the calculation using the closed form does not require calculating values for through , but the recursive form does require calculating values for through , thereby making the recursive form much slower.**